

# Fluctuations and Deconfinement Phase Transition in Nucleus–Nucleus Collisions

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## Abstract

We propose a method to experimentally study the equation of state of strongly interacting matter created at the early stage of nucleus–nucleus collisions. The method exploits the relation between relative entropy and energy fluctuations and equation of state. As a measurable quantity, the ratio of properly filtered multiplicity to energy fluctuations is proposed. Within a statistical approach to the early stage of nucleus-nucleus collisions, the fluctuation ratio manifests a non-monotonic collision energy dependence with a maximum in the domain where the onset of deconfinement occurs.

**1.** Nucleus–nucleus (A+A) collisions at high energies provide a unique opportunity to study properties of strongly interacting matter which at sufficiently high energy density is predicted to exist in a deconfined or quark-gluon-plasma phase. Success of the statistical models to strong interactions [1] suggests that the system created in these collisions is close to thermodynamical equilibrium. Consequently, the properties of the matter are naturally expressed in terms of its equation of state (EoS) which in turn is sensitive to possible phase transitions. Increasing the energy of nuclear collisions, one expects to achieve at the collision early stage higher and higher energy density that at a certain point is sufficient for creation of the quark-gluon plasma. Then, EoS should experience a qualitative change. Observing a clear signal of this change is among main tasks of the whole experimental program of study A+A collisions. The task, however, has appeared rather difficult. It is far not simple to express thermodynamical characteristics at the early stage through the directly measurable quantities. The entropy is of particular interest, as it is believed to be conserved during the expansion of the matter, and several methods to determine it experimentally have been suggested [2–4]. Other observables, which may be sensitive to the EoS of the early stage matter, have been also proposed. Transverse momentum spectra [5], two pion correlations [6], anisotropic flow [7] and strangeness production [8] are discussed in this context.

**2.** The recently measured energy dependence of the pion multiplicity, which is related to the system’s entropy, and kaon (system’s strangeness) production in central Pb+Pb collisions [9,10] show the changes which are consistent with the hypothesis [8,11] that a transient state of deconfined matter is created at the collision energies higher than about 30 A·GeV in fixed target experiments. This conclusion is reached within the Statistical Model of the Early Stage, SMES [8], which assumes creation of the matter (in confined, mixed or deconfined phase) at early stage of the collision according to the maximum entropy principle.

**3.** In this letter we propose a new method of study of EoS which uses the ratio of properly filtered multiplicity and energy fluctuations as directly measurable quantity and refers to SMES [8] as a physical framework. Within this model the ratio is directly related to the

fluctuations of the early stage entropy and energy and thus is sensitive to the EoS of the early stage matter. We show here that the model predicts a non-monotonic energy dependence of the ratio with the maximum where the onset of deconfinement occurs.

**4.** In thermodynamics, the energy  $E$ , volume  $V$  and entropy  $S$  are related to each other through EoS. Thus, various values of the energy of the initial equilibrium state lead to different, but uniquely determined, initial entropies. When the collision energy is fixed the energy, which is used for particle production, still fluctuates. These fluctuations of the inelastic energy are caused by the fluctuations in the dynamical process which leads to the particle production. They are called here the *dynamical* energy fluctuations. Clearly, the *dynamical* energy fluctuations lead to the *dynamical* fluctuations of entropy, and the relation between them is, in the thermodynamical approach, given by EoS. Consequently, simultaneous event-by-event measurements of both the entropy and energy should yield an information on EoS. Since EoS manifests an anomalous behavior in a phase transition region the anomaly should be also visible in the ratio of entropy to energy fluctuations.

**5.** The energy and entropy can be defined in any form of matter, confined, mixed and deconfined, in the collision early stage and in the system's final state. If the produced matter can be treated as an isolated system, the energy is obviously conserved. The entropy is also expected to be conserved during the system's expansion and freeze-out. However, there is a significant difference between the two quantities. While the energy is defined for every event the entropy refers to an ensemble of events.

**6.** Since we are going to discuss the collision energy dependence of the fluctuations within the SMES [8], let us present the model's basic assumptions. The volume,  $V$ , where the matter in confined, mixed or deconfined state is produced at the collision early stage, is given by the Lorentz contracted volume occupied by wounded nucleons. For the most central collisions the number of wounded nucleons  $N_W \approx 2A$ . The net baryonic number of the *created* matter equals zero. Even in the most central A+A collisions, only a fraction of the total collision energy is used for a particle production. The rest is taken away by the baryons which

contribute to the baryon net number.

**7.** The fluctuations occurring in the collision early stage, which are local in coordinate or momentum space, are washed out, at least partially, in the course of temporal evolution of the fireball due to relaxation processes such as particle diffusion, see e.g. [12]. This probably explains why the electric charge fluctuations generated at the QGP phase [13,14], which are significantly smaller than those in the hadron phase, are not seen in the experimental data [15–17]. It should be stressed, however, that the relaxation processes are irrelevant for our considerations as we are interested in the fluctuations of *total* inelastic energy and entropy of the system created at the collision early stage. Because of the exact energy and approximate entropy conservation the fluctuations observed in the final state equal to the early stage fluctuations. We assume here that all produced particles are detected but further we relax this assumption. The inelastic energy deposited in the fireball for the particle production should not be confused with the collision energy. While the former one fluctuates the latter is fixed and it does not fluctuate at all.

**8.** We denote by  $\delta E$  the event-by-event deviations of the energy from its average value  $E$  caused by the dynamical fluctuations which occur in the thermalization process. We assume that  $\delta E \ll E$ . As  $E = \varepsilon V$ , where  $\varepsilon$  is the energy density. One has  $\delta E = V\delta\varepsilon + \varepsilon \delta V$ , i.e. the change of the system's energy is due to the changes of the system's energy density and volume which are considered further as two independent thermodynamical variables. The energy density is usually a unique function of the temperature,  $T$ , but when the system experiences a first order phase transition,  $\varepsilon$  in the mixed phase depends on the relative abundance of each phase.

**9.** According to the first and the second principles of thermodynamics, the entropy change  $\delta S$  is given as  $T\delta S = \delta E + p\delta V$ , which provides  $T\delta S = V\delta\varepsilon + (p + \varepsilon)\delta V$ , where  $p$  is the pressure. Using the identity  $TS = E + pV$  one finds

$$\frac{\delta S}{S} = \frac{1}{1 + p/\varepsilon} \frac{\delta\varepsilon}{\varepsilon} + \frac{\delta V}{V} . \quad (1)$$

**10.** When  $\delta\varepsilon = 0$ , i.e. when the fluctuations of the initial energy and entropy are entirely due to the volume fluctuations at a constant energy density, Eq. (1) provides:  $\delta S/S = \delta V/V = \delta E/E$ . Thus, the relative dynamical fluctuations of entropy are exactly equal to those of energy and they are insensitive to the form of EoS. The  $\delta\varepsilon = 0$  limit may serve as an approximation for all inelastic A+A collisions where fluctuations of the collision geometry dominate all other fluctuations. This case, however, is not interesting from our point of view.

**11.** When  $\delta V = 0$  the fluctuations of the initial energy,  $\delta E$ , are entirely due to the energy density fluctuations. In this case Eq. (1) gives:

$$\frac{\delta S}{S} = \frac{\delta E}{E} \frac{1}{1 + p/\varepsilon} . \quad (2)$$

As seen,  $\delta S/S$  is now sensitive, via the factor  $(1 + p/\varepsilon)^{-1}$ , to the EoS at the early stage of A+A collision. We are interested just in such a situation.

**12.** The number of wounded nucleons can, in principle, be measured on the event-by-event basis. This can be achieved by measuring the number of spectator nucleons,  $N_S$ , in the so-called zero degree calorimeter, used in many experiments. Then,  $N_W \approx 2(A - N_S)$ . Selecting the most central events, we can neglect contribution from the impact parameter variation. Since the system's volume, as defined in SMES, is then fixed the entropy fluctuations are given by Eq. (2).

**13.** To study the entropy fluctuations it appears convenient to introduce the ratio of relative fluctuations:

$$R_e \equiv \frac{(\delta S)^2/S^2}{(\delta E)^2/E^2} = \left(1 + \frac{p}{\varepsilon}\right)^{-2} , \quad (3)$$

which qualitatively behaves as follows. The ratio  $p/\varepsilon$  is about 1/3 in both the confined phase and in the hot quark-gluon plasma (QGP). Then,  $R_e \approx (3/4)^2 \cong 0.56$  and it is rather independent of the collision energy except the domain where the initially created matter experiences the deconfinement phase transition. An exact nature of the transition

is unknown but modelling of the transition by means of the lattice QCD [18] shows a very rapid change of the  $p/\varepsilon$  ratio in a narrow temperature interval  $\Delta T \cong 5$  MeV where the energy density grows by about an order of magnitude whereas the pressure remains nearly unchanged. One refers to this temperature interval as a ‘generalized mixed phase’. The ratio  $p/\varepsilon$  reaches minimum at the so-called softest point of the EoS [6] which corresponds to a maximum of  $R_e \approx 1$ . Consequently, we expect a non-monotonic behavior of the ratio  $R_e$  as a function of the collision energy.

**14.** The energy dependence of the fluctuation ratio  $R_e$  calculated within SMES [8] (using its standard values of all parameters) is shown in Fig. 1. We repeat here that the model correctly reproduces the energy dependence of pion and strangeness production and it relates experimentally observed anomalies to the onset of deconfinement. Within the model, the confined matter, which is modelled as an ideal gas, is created at the collision early stage below the energy of 30 A·GeV. In this domain, the ratio  $R_e$  is approximately independent of collision energy and equals about 0.6. The model assumes that the deconfinement phase-transition is of the first order. Thus, there is the mixed phase region, corresponding to the energy interval 35÷60 A·GeV, where  $R_e$  ratio increases and reaches its maximum,  $R_e \approx 0.8$ , at the end of the transition domain. Further on, in the pure QGP phase represented by an ideal quark-gluon gas under bag pressure, the ratio decreases and  $R_e$  approaches its asymptotic value 0.56 at the highest SPS energy 160A·GeV. Small deviations from  $p = \varepsilon/3$  are in SMES due to non-zero masses of strange degrees of freedom, both in confined and deconfined phases, and due to the bag pressure in QGP. The two effects can be safely neglected at  $T \gg T_c$ .

**15.** In principle, the initial energy fluctuations might be sizable while our analysis holds for infinitesimally small fluctuations as the ratio  $R_e$  (3) is defined above by introducing the dynamical energy fluctuations  $\delta E$  and we use thermodynamical identities to calculate the entropy fluctuations  $\delta S$ . However, the calculations with explicit initial energy distribution show that the finite size of initial energy fluctuations does not much change our results. The

dependence of  $R_e$  on the collision energy shown in Fig. 1 remains essentially the same. The only difference is a ‘smooth’ behavior of  $R_e(F)$  near the maximum.

**16.** The early stage energy and entropy fluctuations are not directly observable, however, as we discuss in the remaining part of the paper,  $R_e$  can be inferred from the experimentally accessible information. Since the energy of an isolated system is a conserved quantity, one measures the initial energy deposited for the particle production, summing up the final state energies of *all* produced particles. The system’s entropy is not strictly conserved but, as already discussed, it is approximately conserved. Therefore, the final state entropy of *all* produced particles is close to the initial entropy. The entropy cannot be directly measured but it can be expressed through measurable quantities.

**17.** As well known, the system’s entropy is related to the mean particle multiplicity. For example,  $\overline{N} = S/3.6$  in the ideal gas of massless bosons. The relation is, in general, more complex but we assume that the final state mean multiplicity is proportional to the initial state entropy, i.e.  $\overline{N} \sim S$ . With the over-bar we denote averaging over events that have identical initial conditions (the same amount of energy is deposited for the particle production). It is clear that for the class of events with a fixed value of  $\overline{N}$ , the multiplicity  $N$  measured in each event fluctuates around  $\overline{N}$ . These are *statistical* but not dynamical fluctuations. We note that particle multiplicity can be determined for every event, in contrast to the entropy which is defined by averaging of hadron multiplicities in the ensemble of events. Since  $\overline{N} \sim S$ , we get:  $\delta S/S = \delta \overline{N}/\overline{N}$ . Thus, the dynamical entropy fluctuations are equal to the dynamical fluctuations of the mean multiplicity. It is crucial to distinguish the dynamical fluctuations of  $\overline{N}$  from the statistical fluctuations of  $N$  around  $\overline{N}$ . We clarify this point below.

**18.** The multiplicity  $N$  measured on event-by-event basis varies not only due to the dynamical fluctuations at a collision early stage but predominately due to the statistical fluctuations at freeze-out. Thus, the final multiplicity distribution,  $\mathcal{P}(N)$ , is given by:

$$\mathcal{P}(N) = \int_0^\infty d\bar{N} W(\bar{N}) P_{\bar{N}}(N) , \quad (4)$$

where  $W(\bar{N})$  describes fluctuations of  $\bar{N}$  due to dynamical fluctuations of  $E$ , and  $P_{\bar{N}}(N)$  is the statistical probability distribution of  $N$  for a given  $\bar{N}$ . The finally measured mean value of an observable  $f(N)$  results from averaging over the  $W$  and  $P$  distributions as

$$\langle\langle f_N \rangle\rangle \equiv \sum_N f(N) \mathcal{P}(N) = \int_0^\infty d\bar{N} W(\bar{N}) \sum_N f(N) P_{\bar{N}}(N) \equiv \langle \overline{f(\bar{N})} \rangle . \quad (5)$$

Thus, the complete averaging,  $\langle\langle \dots \rangle\rangle$ , is done in two steps: first – the statistical,  $\overline{\dots} \equiv \sum_N \dots P_{\bar{N}}(N)$ , and second – the dynamical averaging,  $\langle \dots \rangle \equiv \int_0^\infty d\bar{N} \dots W(\bar{N})$ , one after another. One easily shows that

$$\langle\langle N \rangle\rangle = \langle \bar{N} \rangle , \quad (\Delta N)^2 \equiv \langle\langle N^2 \rangle\rangle - \langle\langle N \rangle\rangle^2 = (\delta \bar{N})^2 + \langle (\delta N)^2 \rangle , \quad (6)$$

where  $(\delta \bar{N})^2 \equiv \langle \bar{N}^2 \rangle - \langle \bar{N} \rangle^2$  and  $(\delta N)^2 \equiv \bar{N}^2 - \bar{N}^2$ . Thus, the total fluctuations  $(\Delta N)^2$ , which are experimentally measured, are equal to the sum of the dynamical (early stage) fluctuations  $(\delta \bar{N})^2$  and the dynamically averaged statistical fluctuations  $\langle (\delta N)^2 \rangle$  at freeze-out.

**19.** We have considered above the ideal detector which measures all produced particles. A real detector, however, measures only a fraction of them, say charged particles in the limited momentum acceptance of the detector. Let us denote the mean energy and multiplicity of accepted particles as  $\bar{E}_A$  and  $\bar{N}_A$ . We assume that

$$\frac{\delta \bar{E}_A}{\bar{E}_A} = \frac{\delta E}{E} , \quad \frac{\delta \bar{N}_A}{\bar{N}_A} = \frac{\delta S}{S} , \quad (7)$$

i.e. relative dynamical fluctuations of the mean energy and mean multiplicity of accepted particles are equal to the relative dynamical fluctuations of the total energy and entropy in the initial state. In our further considerations, we will omit the index ‘A’, however, it is understood that we deal with the accepted particles.

**20.** There is a simple procedure to eliminate the statistical fluctuations, and thus, to extract the dynamical fluctuations of interest from the measured fluctuations, if  $P_{\bar{N}}(N)$  is



the Poisson distribution. Then,  $(\delta N)^2 = \overline{N}$ , and  $(\delta \overline{N})^2 = (\Delta N)^2 - \langle \langle N \rangle \rangle$ . Therefore, the relative dynamical fluctuations are expressed through the total relative fluctuations as

$$\left( \frac{\delta \overline{N}}{\langle \langle N \rangle \rangle} \right)^2 = \left( \frac{\Delta N}{\langle \langle N \rangle \rangle} \right)^2 - \frac{1}{\langle \langle N \rangle \rangle} . \quad (8)$$

The distribution of energy  $E$  of the system of several particles is assumed to be of the form

$$\mathcal{P}(E) = \sum_N \int d\zeta W(\zeta) P_\zeta(N) \int d\omega_1 P_\zeta(\omega_1) \cdots \int d\omega_N P_\zeta(\omega_N) \delta(E - \sum_{i=1}^N \omega_i) , \quad (9)$$

where  $W(\zeta)$  describes dynamical fluctuations of the parameter  $\zeta$  which controls the multiplicity and energy fluctuations. In principle,  $\zeta$  can be understood as a whole set of parameters.  $P_\zeta(N)$  is the multiplicity and  $P_\zeta(\omega)$  single particle energy distribution, both giving the statistical fluctuations. One easily finds that

$$\langle \langle E \rangle \rangle = \langle \overline{N} \overline{\omega} \rangle , \quad (10)$$

$$(\Delta E)^2 \equiv \langle \langle E^2 \rangle \rangle - \langle \langle E \rangle \rangle^2 = (\delta \overline{E})^2 + \langle (\delta E)^2 \rangle , \quad (11)$$

where  $\overline{\omega^n} \equiv \int d\omega \omega^n P_\zeta(\omega)$  and

$$(\delta \overline{E})^2 \equiv \langle \overline{E^2} \rangle - \langle \overline{E} \rangle^2 = \langle (\overline{N} \overline{\omega})^2 \rangle - \langle \overline{N} \overline{\omega} \rangle^2 , \quad (12)$$

$$\langle (\delta E)^2 \rangle \equiv \langle \overline{E^2} - \overline{E}^2 \rangle = \langle \overline{N}(\overline{\omega^2} - \overline{\omega}^2) \rangle + \langle (\overline{N^2} - \overline{N}^2) \overline{\omega}^2 \rangle . \quad (13)$$

One sees that  $\delta \overline{E} = 0$  for vanishing dynamical fluctuations *i.e.* when  $W(\zeta) = \delta(\zeta - \zeta_0)$ .

Assuming again that the multiplicity distribution  $P_\zeta(N)$  is poissonian, then  $\overline{N^2} - \overline{N}^2 = \overline{N}$ , and  $\langle (\delta E)^2 \rangle$  reads

$$\langle (\delta E)^2 \rangle = \langle \overline{N} \overline{\omega^2} \rangle = \langle \langle N \rangle \rangle \int d\omega \omega^2 P_{\text{incl}}(\omega) , \quad (14)$$

where  $P_{\text{incl}}(\omega)$  is the single particle inclusive energy distribution defined as

$$P_{\text{incl}}(\omega) \equiv \frac{1}{\langle \langle N \rangle \rangle} \sum_N N \int d\zeta W(\zeta) P_\zeta(N) P_\zeta(\omega) . \quad (15)$$

Thus, the relative dynamical fluctuations of energy equal

$$\left( \frac{\delta \overline{E}}{\langle \langle E \rangle \rangle} \right)^2 = \left( \frac{\Delta E}{\langle \langle E \rangle \rangle} \right)^2 - \frac{\lambda}{\langle \langle N \rangle \rangle} , \quad (16)$$

where

$$\lambda \equiv \frac{\int d\omega \omega^2 P_{\text{incl}}(\omega)}{\left( \int d\omega \omega P_{\text{incl}}(\omega) \right)^2} . \quad (17)$$

**21.** In general, the statistical fluctuations are not poissonian, and *a priori* their form is even not known. The dynamical fluctuations can be then measured by means of the so-called sub-event method [19] where one considers two different, non-overlapping but dynamically equivalent regions of the momentum space ‘1’ and ‘2’. These can be two equal to each other non-overlapping rapidity intervals symmetric with respect to the center-of-mass rapidity. Let  $N_1$  and  $N_2$  are the numbers of hadrons (e.g. negative pions) in these regions. There is a principal difference between the dynamical and statistical fluctuations discussed above. The statistical event-by-event fluctuations of  $N_1$  and  $N_2$  in different parts of the momentum space are uncorrelated:  $P(N_1, N_2) = P_1(N_1) \cdot P_2(N_2)$ . The dynamical fluctuations represent, according to Eq. (7), a correlated change of the average particle numbers  $\overline{N}_1$  and  $\overline{N}_2$  with that of total entropy. Since these average values are equal to each other,  $\overline{N}_1 = \overline{N}_2 \equiv \overline{N}$  (the regions ‘1’ and ‘2’ are dynamically equivalent), the distributions of statistical fluctuations are also the same:  $P_1(N_1) \equiv P_{\overline{N}}(N_1)$  and  $P_2(N_2) \equiv P_{\overline{N}}(N_2)$ . Therefore, the total probability for detecting  $N_1$  particles in the region ‘1’ and  $N_2$  particles in the region ‘2’ is

$$\mathcal{P}(N_1, N_2) = \int_0^\infty d\overline{N} W(\overline{N}) P_{\overline{N}}(N_1) \cdot P_{\overline{N}}(N_2) , \quad (18)$$

and the total averaging of an observable  $f(N_1, N_2)$  provides:

$$\begin{aligned} \langle\langle f(N_1, N_2) \rangle\rangle &\equiv \sum_{N_1, N_2} f(N_1, N_2) \mathcal{P}(N_1, N_2) \\ &= \int_0^\infty d\overline{N} W(\overline{N}) \sum_{N_1, N_2} f(N_1, N_2) P_{\overline{N}}(N_1) \cdot P_{\overline{N}}(N_2) . \end{aligned} \quad (19)$$

It follows from Eq. (19) that

$$\frac{1}{2} \langle\langle (N_1 - N_2)^2 \rangle\rangle = \langle\overline{N}^2\rangle - \langle\overline{N}\rangle^2 \equiv \langle(\delta N)^2\rangle . \quad (20)$$

Therefore, measuring the total fluctuations of  $(N_1 - N_2)/2$ , one obtains the dynamically averaged statistical fluctuations in the region ‘1’ (equal to that in the region ‘2’). Subtracting  $\langle(\delta N)^2\rangle$  from the total fluctuations in this region,  $(\Delta N)^2$ , one finds the dynamical

part,  $(\delta\overline{N})^2$ , of interest. Similar analysis can be performed to get the dynamical energy fluctuations.

**22.** We have assumed that only dynamical fluctuations generated at the collision early stage lead to the particle correlations in the final state. Of course, it is not quite true. The effects of quantum statistics also lead to the inter-particle correlations. However, the correlation range in the momentum space is in this case rather small,  $\Delta p \approx 100 \text{ MeV}/c$ . The contribution of these effects can be accounted in  $\langle(\delta N)^2\rangle$  if the selected acceptance regions are separated by the distance significantly larger than  $\Delta p$ .

**23.** There are also long range correlations which have nothing to do with the early stage dynamical correlations and cannot be accounted in  $\langle(\delta N)^2\rangle$  by the sub-event method described above. In particular, there are correlations due to conservation laws. Those can be effectively eliminated if one studies only a small part of a whole system which is constrained by the conservation laws.

**24.** A large fraction of the final state particles comes from the decays of various hadron resonances. The existence of resonances decaying into at least two hadrons enlarges the final state multiplicity fluctuations. This effect can not be eliminated by use of the sub-event method. It is because the decay products are correlated at the scale of approximately one rapidity unit which at the SPS energy domain is comparable to the width of rapidity distribution. To remove bias due to resonance production and decay, we suggest to study the fluctuations of negatively charged hadrons as typically only one negatively charged hadron comes from a single resonance decay.

**25.** In summary, we propose a new method to study the equation of state of strongly interacting matter produced at the early stage of nucleus–nucleus collisions. The method exploits the properly filtered relative fluctuations of multiplicity and energy. Within the Statistical Model of the Early Stage [8] this ratio is directly related to the fluctuations of the early stage entropy and energy and thus is sensitive to the EoS of the early stage matter. We show that within the model the ratio is a non–monotonic function of the collision energy

with the maximum at the end of the mixed phase ( $\approx 60$  A·GeV). Consequently, it can be considered as a further signal of deconfinement phase transition.

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# FIGURES

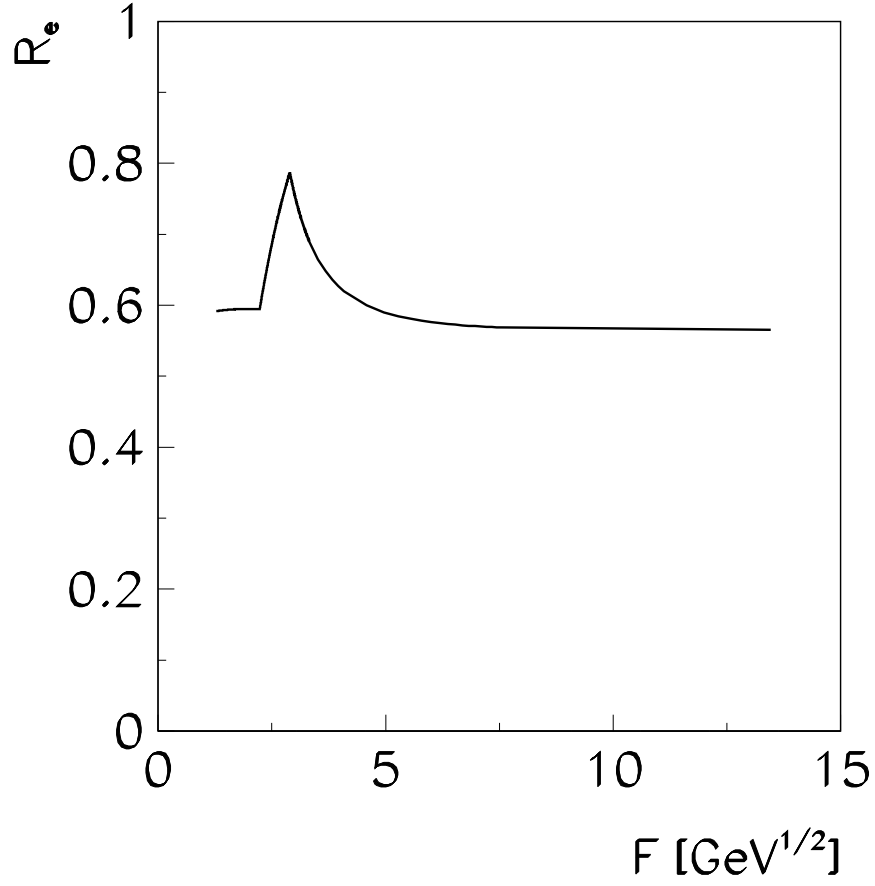


FIG. 1. The dependence of  $R_e$  calculated within SMES [8] on the Fermi's collision energy measure  $F \equiv (\sqrt{s} - 2m)^{3/4}/s^{1/8}$  where  $\sqrt{s}$  is the c.m.s. energy per nucleon–nucleon pair and  $m$  is the nucleon mass. The ‘shark fin’ structure is caused by the large fluctuations in the mixed phase region.